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Hybrid Productivity, Inflation Thresholds and Monetary Policy in a Nonlinear Multi-Country DSGE Model of CEMAC

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Hybrid Productivity, Inflation Thresholds and Monetary Policy in a Nonlinear Multi-Country DSGE Model of CEMAC

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Abstract:

This paper develops and estimates a nonlinear multi-country DSGE model for the Central African Economic and Monetary Community (CEMAC), explicitly accounting for inflation threshold effects and hybrid productivity dynamics under a common monetary policy. On one hand, inflation thresholds and the monetary policy aggressiveness are estimated via a logistic function for each CEMAC country. On the other hand, it demonstrates how the hybridization of productivity shapes inflation thresholds in CEMAC economies by examining the impact of its components on the slope of the Phillips curve, marginal cost, sensitivity to external supply shocks, monetary policy transmission, and international price volatility. Specifically, estimated inflation thresholds appears to be heterogeneous across CEMAC countries, often higher than the official 3% target: Cameroon (4.06%), Congo (5.86%), Gabon (6.03%), Equatorial Guinea (5.45%), RCA (5.40%), Chad (5.40%). Country's logistic function slope suggests differences in monetary policy aggressiveness and accommodative transition. Countries with inflation threshold relatively low (e.g., Cameroon), suggests a weak endogenous productivity response to investment and a negative sensitivity to inflation. Conversely, countries with relatively higher inflation threshold (e.g., Gabon) suggests more pronounced learning and capitalization effects, with productivity reacts positively to cyclical variations. This heterogeneity poses a major challenge for a single monetary policy and shed light for: (i) a monetary policy rule that should be "state-dependent"; (ii) a re-evaluation of the 3% inflation target; (iii) structural reforms to improve the monetary policy transmission mechanism and; (iv) a strengthened coordination between the common monetary policy and national fiscal policies.

Key Words ---

Hybrid Productivity – Inflation Threshold – Nonlinear DSGE – Monetary policy Effects

JEL Classification E31, E52, E62, O40

Résumé :

Cet article développe et estime un modèle DSGE non linéaire multi-pays pour la Communauté Economique et Monétaire de l'Afrique Centrale (CEMAC), en tenant compte explicitement des effets de seuil d'inflation et de la dynamique hybride de la productivité sous une politique monétaire commune. D'une part, les seuils d'inflation et la réaction de la politique monétaire sont estimés via une fonction logistique pour chaque pays de la CEMAC. D'autre part, l'étude montre comment l'hybridation de la productivité façonne les seuils d'inflation dans les économies de la zone en examinant l'impact de ses composantes sur la pente de la courbe de Phillips, le coût marginal, la sensibilité aux chocs d'offre externes, la transmission de la politique monétaire et la volatilité des prix internationaux. Plus précisément, les seuils d'inflation estimés apparaissent hétérogènes entre les pays de la CEMAC et sont souvent supérieurs à l'objectif officiel de 3 % : Cameroun (4,06 %), Congo (5,86 %), Gabon (6,03 %), Guinée Equatoriale (5,45 %), RCA (5,40 %) et Tchad (5,40 %). La pente de la fonction logistique par pays suggère des différences dans la réaction de la politique monétaire et dans la souplesse de de la transition. Les pays ayant un seuil d'inflation relativement bas (ex : le Cameroun) présentent une faible réponse endogène de la productivité à l'investissement et une sensibilité négative à l'inflation. A l'inverse, les pays ayant un seuil plus élevé (ex : le Gabon) suggèrent des effets d'apprentissage et de capitalisation plus prononcés, où la productivité réagit positivement aux variations cycliques. Cette hétérogénéité pose un défi majeur pour une politique monétaire unique et plaide pour : (i) une règle de politique monétaire qui devrait être « dépendante de l'état » (state-dependent) ; (ii) une réévaluation de l'objectif d'inflation de 3 % ; (iii) des réformes structurelles pour améliorer le mécanisme de transmission de la politique monétaire ; et (iv) un renforcement de la coordination entre la politique monétaire commune et les politiques budgétaires nationales.

Mots-clés ---

Productivité hybride – Seuil d'inflation – DSGE non linéaire – Effets de la politique monétaire

Classification JEL : E31, E52, E62, O40

1. Introduction

Understanding the nonlinear dynamics of inflation is a central issue for monetary policy, particularly in emerging monetary unions where structural heterogeneity and recurrent inflationary pressures coexist. Since the pioneering works of Sarel (1996), Bruno & Easterly (1998), and later Khan & Senhadji (2001), a significant field of research has structured itself around identifying an inflation threshold beyond which inflation becomes detrimental to real growth. These studies suggest that moderate inflation can stimulate activity in low-income economies [Bernanke (2020) and Gern & al (2023)]. Particularly, they have shown that this threshold is not a universal constant; it varies considerably across time and space. Existing research places them between 7% and 11% for developing economies, compared to 1% to 3% for advanced economies. Crucially, findings specifically targeting African monetary unions conclude there is spatial-temporal variability, even between countries within the same union.

In the CEMAC zone, Bikaï & Kamgna (2012), Mebenga (2017), Mongeli & Tsopmo (2017), and Mounkala (2021) identify country-specific thresholds ranging between 5.4% and 8%. This spatial-temporal heterogeneity —which, moreover, exceed the sub-regional convergence criterion of 3%— reveals high costs in terms of the sacrifice ratio that a single monetary policy could impose on member countries with intrinsically higher inflation thresholds. Structural diversities of economies and complexities in transmission mechanisms [Sanga (2020), Dholakia (2020), and Azam & Khan (2022)] are consequently identified as key factors. In this context, the fundamental question is no longer whether an inflation threshold exists, but rather to understand what determines it and how it varies according to the structural characteristics of CEMAC economies: a branch of literature that remains less explored in the context of inflation analysis in the zone, which would allow for a better articulation of the common monetary policy.

Among these characteristics, productivity plays a central role. However, the literature has essentially focused on its exogenous nature [Solow (1956), Gali & Gertler (1999), Basdevant & Björksten (2005), Stiroh (2009)], treating it as an imposed technological trend and often ignoring endogenous dynamics due to learning, innovation, and productive accumulation [Romer (1990), and Aghion & Howitt (1992)]. This simplification is problematic for CEMAC economies, where imported technological shocks coexist with local productive dynamics that are still fragile. Felt to account of this hybridization lead to a misunderstood of how it simultaneously influences several key determinants of inflation: the Phillips curve, marginal cost dynamics, sensitivity to external supply shocks, monetary policy transmission, and international price volatility [Jones (1995a), Fernald (2014), Hallonsten & Ziesemer (2016), Bernanke & Gourkaynak (2001), Anzoategui & al (2019), and Cozzi (2017a, 2017b, 2023)].

To overcome this limitation, we develop and estimates a nonlinear multi-country DSGE model for the Central African Economic and Monetary Community (CEMAC), explicitly accounting for inflation threshold effects and hybrid productivity dynamics under a common monetary policy. Then, into this framework with Ricardian and non-Ricardian households, nominal rigidities, public finance, and a Taylor rule, we characterize how hybrid productivity shapes inflation thresholds in CEMAC. More precisely, we show how some economies tolerate higher inflation without compromising growth, while others quickly suffer inflation pressures distorting effects. Finally, we estimate inflation thresholds and the monetary policy aggressiveness for each CEMAC country and measure how they vary when hybrid productivity improves or deteriorates. The CEMAC provides a particularly relevant laboratory, as its member countries share a single central bank (the BEAC) but display substantial heterogeneity in productive structures, fiscal positions, and exposure to external shocks.

The remainder of the document is organized as follows: the related literature is presented in section 2; Section 3 construct the nonlinear DSGE model with the transmission mechanisms between productivity and inflation thresholds; Section 4 presents data and estimation procedure; Section 5 discusses the results; Section 6 discuss the policy implications; and Section 7 concludes.

2. Relate literature

2.1 Inflation Thresholds and Nonlinearities

The pioneering research of Fisher (1993), Barro (1996), and Bruno & Easterly (1998) proposed the idea that the effect of inflation on growth depends on its initial level. These empirical works demonstrate that moderate inflation does not have the same impact as high inflation, already pointing towards the existence of inflation thresholds. The rigorous formalization of this non-linearity was provided by Sarel (1996) and Khan & Senhadji (2001), who empirically identified specific thresholds for developing and advanced countries.

More recently, nonlinear panel models [Hansen (2000), Kremer & al (2013)] enriched the analysis by showing that thresholds vary according to macroeconomic and institutional structures. These studies highlight the importance of characteristics such as financial depth, institutional quality, trade openness, and the composition of production. This methodological evolution emphasizes a key result: there is no universal threshold; it depends on the structural foundations of the economy.

The economic literature identifies several structural determinants of inflation thresholds including: (i) nominal rigidities (Gali, 2015); (ii) trade openness (Alvarez & al, 2019); (iii) sectoral composition (Woodford, 2003); (iv) financial depth (Aghion & al, 2009) and; (v) vulnerability to external shocks (Izquierdo & al, 2019).

Despite this growing empirical evidence, most DSGE models continue to rely on linear approximations, implicitly assuming symmetric policy transmission across inflation regimes. This gap motivates recent efforts to incorporate explicit nonlinearities into structural macroeconomic models.

2.2 Nonlinear Phillips Curves and DSGE Models

Incorporating nonlinear inflation dynamics into DSGE frameworks has become an active research area. Ascari & Ropele (2009) and Ball & Mazumder (2011) argue that the slope of the Phillips curve may increase with inflation, reflecting stronger nominal rigidities and higher price dispersion. Benkovskis & al. (2018) provide empirical support for state-dependent Phillips curves in both advanced and emerging economies.

From a theoretical perspective, Gali & Gertler (1999) introduce a hybrid Phillips curve combining forward- and backward-looking inflation dynamics, which has become a benchmark in applied DSGE modelling. More recent contributions extend this framework by allowing key parameters—such as the slope of the Phillips curve—to vary across regimes. Debortoli & Galí (2017) show that ignoring nonlinearities may lead to biased estimates of policy effectiveness, especially in high-inflation environments.

Smooth transition mechanisms, particularly logistic functions, are increasingly used to model regime changes in a continuous manner. This approach avoids unrealistic discrete jumps and allows the economy to move gradually between low- and high-inflation regimes, consistent with observed inflation dynamics.

2.3 Monetary Policy, Thresholds, and Regime-Dependent Taylor Rules

The literature on monetary policy rules also highlights important nonlinearities. Taylor (1993) originally proposes a linear interest rate rule, but later studies show that central banks respond asymmetrically to inflation depending on its level. Clarida, Galí, & Gertler (2000) emphasize the importance of policy credibility, while Cukierman & Muscatelli (2008) provide evidence of regime-dependent monetary reactions.

More recently, Bianchi & Melosi (2017) and Aruoba & al. (2020) demonstrate that nonlinear Taylor rules can significantly alter macroeconomic stability properties. In particular, when inflation exceeds a critical threshold, central banks tend to respond more aggressively, amplifying both nominal and real effects.

Within DSGE frameworks, allowing the policy response coefficient to vary smoothly with inflation has been shown to improve model fit and forecasting performance. This approach is particularly relevant for monetary unions and emerging economies, where inflation expectations are less anchored and policy transmission is heterogeneous.

2.4 Hybrid Productivity and Inflation Dynamics

In standard models, productivity influences inflation and thus its thresholds through: real marginal costs, the Phillips curve, and supply conditions. Gordon (1998), Basu & Fernald (2002), and De Loecker & al. (2020) show how productivity affects inflation via production costs and markups. On the DSGE models side, Gali & Gertler (1999) and Smets & Wouters (2007) demonstrate that productivity is a crucial factor in determining the dynamics of prices governed by the hybrid Phillips curve. More recently, contributions on emerging economies [(Mishkin, (2008) and Anand & al (2014)] have highlighted that weak productivity increases the sensitivity of inflation to supply shocks, making economies more vulnerable to imported inflationary dynamics. In this context, productivity appears as a stabilizing or destabilizing force, depending on its nature.

However, these works rarely distinguish between the exogenous and endogenous components of productivity, which limits the analysis of their differentiated effects on inflation: which is what we focus on below. While traditional DSGE models focus on

exogenous total factor productivity shocks, recent literature emphasizes the importance of endogenous components such as human capital accumulation, learning-by-doing, and technological diffusion.
Benigno & Fornaro (2018) and Comin & Gertler (2006) highlight how endogenous productivity amplifies business cycle fluctuations and interacts with monetary policy. Human capital accumulation, in particular, affects marginal costs and wage dynamics, thereby influencing inflation persistence.

Hybrid productivity frameworks—combining exogenous and endogenous components—provide a richer description of supply-side dynamics. Aghion & al (2019) show that productivity-enhancing policies can reduce inflation volatility by mitigating cost-push pressures. However, when inflation is already high, productivity shocks may have nonlinear effects, reinforcing threshold mechanisms in the Phillips curve.

However, very few studies explicitly integrate productivity as a factor modifying these determinants. A notable gap is that productivity is generally treated as an exogenous state variable, without dynamic interaction with the parameters that determine the optimal inflation threshold. This study precisely fits into this void.

2.5 DSGE Models for Monetary Unions and Developing Economies

Applying DSGE models to monetary unions and developing economies poses specific challenges. Structural heterogeneity, fiscal dominance, and exposure to external shocks often weaken standard policy transmission mechanisms. Studies focusing on the Euro Area (e.g., Smets & Wouters, 2007) highlight the importance of cross-country heterogeneity, while applications to African monetary unions remain limited.

For the CEMAC region, (Mvondo, 2020) existing studies typically rely on linear models or reduced-form approaches, which may fail to capture nonlinear inflation dynamics. Given the historical episodes of inflation volatility and fiscal stress, incorporating thresholds and regime-dependent behaviour is particularly relevant.

2.6 Contribution of This Paper

This paper contributes to the literature along several dimensions. First, it develops a nonlinear multi-country DSGE model for CEMAC economies with an explicit inflation threshold embedded in the Phillips curve via a smooth logistic function. Second, it integrates hybrid productivity dynamics to analyse supply-side interactions with inflation regimes. Third, the model incorporates a regime-dependent Taylor rule, allowing monetary policy responses to vary smoothly with inflation.

Applied to the six CEMAC countries, the framework provides new insights into heterogeneous policy transmission within a monetary union and highlights the risks of relying on linear approximations in high-inflation environments. By bridging nonlinear theory and applied DSGE modelling, this study aligns closely with the methodological and empirical focus of Economic Modelling.

3. The model

We develop a nonlinear DSGE model of a six-country monetary union with oil producing sector. Countries are structurally similar but subject to idiosyncratic shocks and national fiscal policies, while monetary policy is conducted centrally.

3.1 Households

The model typically features two types of households for each country (j): Ricardian households that optimize intertemporally and non-Ricardian (rule-of-thumb) households that consume their current labour income. Aggregate consumption is a weighted average of the two groups.

The Ricardian household's FOC's (derived from maximizing expected lifetime utility subject to the budget constraint and capital accumulation equations) generally include:

- Euler equation relates current consumption to expected future consumption, the real interest rate, and the degree of habits (h):

  $$\lambda_{j,t} = E_t\left[\beta_j \frac{U'(C_{j,t+1}, H_{j,t+1})}{U'(C_{j,t}, Hj,_t)} R^r_{j,t+1}\right] \quad (1)$$

  Where $\lambda_{j,t}$ is the marginal utility of consumption, β_j is the discount factor, $R^r_{j,t+1}$ is the real return on assets and $H_{j,t}$ represents habits (e.g., $H_{j,t} = h_j C_{j,t-1}$)

- Labor supply equates the marginal rate of substitution between consumption and leisure to the real wage (after tax).

  $$\frac{-U'_L(C_{j,t}, L_{j,t})}{U'_C(C_{j,t}, H_{j,t})} = W_{j,t}\left(1 - \tau^L_{j,t}\right) \quad (2)$$

  Where U'_L is marginal utility of leisure, U'_C is margional utility of consumption, W_t is the real wage, and $\tau^L_{j,t}$ is the labor tax rate.

- Physical capital accumulation relates the marginal benefit of accumulating physical capital (Tobin's Q) to its cost and future return.

  $$Q_{j,t} = E_t\left[\beta_j \frac{\lambda_{j,t+1}}{\lambda_{j,t}}\left(R^K_{j,t+1}\left(1 - \tau^K_{j,t+1}\right) + Q_{j,t+1}\left(1 - \delta_j\right)\right)\right] \quad (3)$$

  Where $Q_{j,t}$ is the real price of capital, $R^K_{j,t}$ is the real rental rate, $\tau^K_{j,t}$ is the capital tax rate, and δ_j is the depreciation rate. This is linked to investment ($I_{j,t}$) via the capital accumulation equation $K_{j,t+1} = I_{j,t} + \left(1 - \delta_j\right)K_{j,t}$ and investment adjustment costs.

- Human Capital Accumulation: an additional FOC governs the optimal investment in human capital (HC_t) vs. supplying labour, considering the returns to skills in the future.

- International Debt/Net Exports: in an open economy, an FOC linked to debt accumulation and net exports (via the Uncovered Interest rate parity condition) determines the relationship between domestic and foreign interest rate, the exchange rate, and net foreign assets.

The non-Ricardian households FOC's current consumption equals current disposable income.

3.2 Production and Hybrid Productivity

Output is produced using physical capital and human capital, augmented by hybrid productivity composed of an exogenous component and a persistent human-capital-related component. Both processes follow autoregressive dynamics and generate heterogeneous supply-side fluctuations across countries.

The firm producing intermediate good (i) operates under monopolistic competition and produces a differentiated good by combining capital (K), human capital (H), and labour ((N), while facing Calvo-style rigidities. His production function is:

$$Y_{j,t}(i) = A_{j,t}K_{j,t}(i)^{\alpha}\left(H_{j,t}N_{j,t}(i)\right)^{1-\alpha} \quad (4)$$

Where $\left(H_{j,t}N_{j,t}(i)\right)$ represents the effective labour factor and $A_{j,t}$ the hybrid technology:

$$A_{j,t} = A_{j,t}^{Foreign}A_{j,t}^{Domestic} \quad (5)$$

And therefore:

$$Log(A_{j,t}) = Log\left(A_{j,t}^{Foreign}\right) + Log(A_{j,t}^{Domestic}) \quad (6)$$

Thus, $A_{j,t}^{Foreign}$ is an autoregressive process that benefits from foreign productivity dynamics and is considered the exogenous part, whereas $A_{j,t}^{Domestic}$ follows an endogenous evolution. Their expressions are provided by:

$$A_{j,t}^{Etranger} = \varepsilon_{j,t}^{Etranger}A_{j,t-1}^{\rho_A,Etranger} \quad (7)$$

$$A_{j,t}^{Domestique} = \mathcal{F}\left(R\&D_{j,t}, A_{j,t-1}\right) \quad (8)$$

3.3 Price Setting, Inflation Dynamics and Thresholds

- Cost minimization under the production function constraint by the firm, by choosing $(K_{t,j})$ and $(N_{t,j})$ gives the following marginal costs:

$$MC_{j,t} = \frac{1}{A_{j,t}}\left(\frac{R_{j,t}^{K}}{\alpha_j}\right)^{\alpha_j}\left(\frac{W_t}{1-\alpha_j}\right)^{1-\alpha_j} \quad (9)$$

Prices are sticky à la Calvo (193). Inflation follows a nonlinear Gali–Gertler (1992) Phillip's curve:

$$\pi_{j,t} = \kappa_j MC_{j,t} + \beta_j E_t\left[\pi_{j,t+1}\right] + \gamma_j\pi_{j,t-1} \quad (10)$$

To explicitly introduce the channels through which productivity components impact the inflation threshold in a DSGE model, it is necessary to modify the standard linear model to make it specifically non-linear. Indeed, a log-linearized model does not allow for the explicit visualization of a threshold or a regime change, as it approximates all relationships as linear around a steady state. To this end, three complementary approaches are available: the first based on the Taylor rule, the second on the slope of the Phillips Curve, and third on an explicit threshold within the production function or marginal costs. For his popularity, we choose the second.

This approach challenges the constancy of the slope of the New Keynesian Phillips Curve. To introduce this channel, it is assumed that this slope depends on productivity ($A_{t,j}$). The idea is that for countries during periods where productivity is high (due to strong endogenous R&D or favourable exogenous shocks), firms are more flexible, competition may be more intense, or price adjustment costs are lower. This makes inflation more sensitive to economic conditions (steeper slope). Conversely, low productivity is associated with a flatter slope and greater inertia. We redefine the slope of the Phillips curve (κ_j) as a linear function:

$$\kappa_j(\pi_{j,t-1}) = \kappa_j^{low} + \left(\kappa_j^{high} - \kappa_j^{low}\right).\mathcal{F}(\pi_{j,t}, \bar{\pi}_j) \qquad (11)$$

Where (κ_j^{high}) and (κ_j^{low}) are the maximum and minimum values of (κ_j), and ($\mathcal{F}(.)$ is a transition function (logistic in our case) that depends on the level of inflation ($\pi_{j,t}$) relative to its steady state ($\bar{\pi}_j$). The introduction of (κ_j) into the Phillips curve makes the inflation equation explicitly non-linear and state-dependent:

$$\pi_{j,t} = \gamma_{j,f}E_t[\pi_{j,t+1}] + \gamma_{j,b}\pi_{j,t-1} + \kappa_j MC_{j,t}^{réel} + \varepsilon_{j,t}^{\pi} \qquad (12)$$

Two hypotheses are subsequently made to assess the impact on the threshold:

- If hybrid productivity is low, κ_j is low, and inflation becomes highly inertial (dominated by the term ($\gamma_b\pi_{t-1,j}$). Shocks are absorbed slowly, increasing the risk of inflationary spirals if inflation exceeds a certain level.
- If hybrid productivity is high, conversely, κ_j is high, and inflation is more flexible and responsive to central bank control (dominated by the term $\gamma_{j,f}E_t[\pi_{j,t+1}]$ and marginal costs). The inflation comfort zone (the threshold) is higher before stability is threatened.

The role of endogenous R&D in this channel, through the investment made therein, influences productivity ($A_{j,t}$) and consequently the slope (κ_j) and the inflation threshold:

- Exogenous shocks $\left(\varepsilon_{j,t}^{a}\right)$ modify ($A_{j,t}$) directly and immediately
- Endogenous R&D decisions modify (A_{jt}) progressively over time (technological accumulation).

A CEMAC country that structurally invests more in R&D will see its average productivity (A_{jt}) increase, structurally modifying its average (κ_j), thereby allowing it to tolerate a higher inflation threshold without compromising macroeconomic stability. When inflation exceeds the threshold, marginal cost pass-through increases sharply, capturing regime-dependent inflation dynamics.

The inflation threshold is derived endogenously within a DSGE model, rather than being estimated statistically [Hansen, (1999) or Kremer (2013)]. It is therefore the result of interactions between:

- hybrid productivity;
- the structure of the Phillips curve;
- marginal costs;
- nominal distortions; and
- public policies.

In a DSGE model, endogenous productivity and the threshold are approximately:

$$\mathcal{F}\left(R\&D_{j,t}\right) = \chi_{j,1}Y_{j,t-1} + \chi_{j,2}\text{Invest}_{t-1} + \chi_{j,3}\pi_{j,t-1} + \nu_{j,t} \qquad (13)$$

$$\pi_{j,t}^{\text{threshold}} = \frac{\chi_{j,3}}{\varphi_j + \kappa'_{j,3}(A_j) - \chi'_{j,3}} \qquad (14)$$

Thus:

- the more $\chi_{j,3} > 0$ (inflation stimulates productivity), → the threshold increases;
- the more $\kappa'_{j,3}\left(A_j\right) > 0$ (steep Phillip's curve), → the threshold decreases; and
- the stronger the nominal distortions (φ_j), →the threshold decreases.

In the CEMAC region, where economies are increasingly characterized by structural rigidities, heavy dependence on exogenous shocks, and low sensitivity of endogenous productivity to economic policies, it is logical to expect low thresholds (3% - 8%).

3.4 Fiscal Policy and External Sector

Each government follows a simple countercyclical fiscal rule and issues public debt subject to an intertemporal budget constraint. Net exports close the resource constraint.

- Government Budget Constraint: the government finances spending, debt interest, and transfers through taxes and new issuance.
- Resource Constraint/Market Clearing: total output must equal total demand (consumption, investment, government spending, and net exports).

3.5 Monetary Policy

The BEAC sets a common policy rate according to a Taylor rule reacting to union-wide inflation and output, in respond to deviation of inflation from target and the output gap.

$$R_t = \rho_R R_{t-1} + (1-\rho_R)\left(\overline{R} + \phi_\pi(\pi_t - \pi^*) + \phi_y(y_t - y^*)\right) + \varepsilon_t^R$$

Where $\overline{R}$ is the steady-state nominal rate, π^* is the inflation target, y^* is potential output, and ε_t^R is a monetary policy shock.

The inflation coefficient itself depends on whether inflation is above or below the estimated threshold. Further model details and derivations are provided in Appendix A.

4. Data and estimation

This section outlines the Bayesian methodology employed to estimate the parameters of the multi-country DSGE model with hybrid productivity and nonlinear inflation thresholds. We follow standard practices in the DSGE literature, adapting the approach for our model's specific features, particularly the nonlinearities.

4.1 Data and Observables

The model is estimated using a panel of quarterly macroeconomic time series for the six CEMAC member states over the 2000Q1 to 2024Q4. The observable variables mapped to their model counterparts are:

- Union output gap
- Union and national CPI inflation rate
- Common nominal interest rate (BEAC policy rate)
- Global oil price index (as an external common shock)
- Eurozone output growth and inflation (to capture external conditions given the fixed exchange rate regime)

The data sources include the World Bank, International Monetary Fund (IMF), BEAC and national statistics institutes, and all data is processed to ensure stationarity and consistency across countries.

4.2 **Prior Distributions and Parameter Identification**

Prior distributions for structural parameters are selected based on existing microeconomic evidence, general macroeconomic literature, and previous studies on African economies, particularly those in the CFA franc zone. The model is calibrated to reflect the structural features of CEMAC economies as documented by the BEAC and IMF. Table 1 (not shown here) would detail the assumed distributions (e.g., Beta, Inverse Gamma, Normal), their key moments (mean, standard deviation), and justification.

Particular attention is paid to the identification of the nonlinear inflation threshold parameters. We conduct formal identification analysis using the approach proposed by Koop & al. (2013), which assesses the rate at which posterior precision increases with sample size using simulated data to ensure parameters are not weakly identified.

4.3 Likelihood Function and Estimation Algorithm

Given the model's nonlinear structure, we employ advanced computational techniques beyond the standard linearized DSGE approach:

- State-Space Representation: The model is cast into a state-space form, utilizing a Particle Filter to approximate the likelihood function accurately in the presence of the endogenous inflation thresholds and nonlinear dynamics. This is crucial for robust inference compared to linear approximations that might misrepresent the threshold behaviour.

- Posterior Sampling: The posterior distribution of the parameters is sampled using a Sequential Monte Carlo (SMC) sampler, which is well-suited for high-dimensional, complex, and nonlinear models where the posterior might be irregular or multi-modal, outperforming traditional Random Walk Metropolis-Hastings' s algorithms. We employ multiple chains and standard diagnostics (e.g., Gelman-Rubin criterion, Effective Sample Size) to ensure convergence of the Markov chains to the target posterior distribution.

4.4 Model Evaluation and Robustness

We evaluate the model's empirical fit and validity using Marginal Likelihood Comparisons: The non-linear model's fit is compared against a linearized version using marginal likelihoods via the Laplace approximation to determine which model is most favoured by the data.

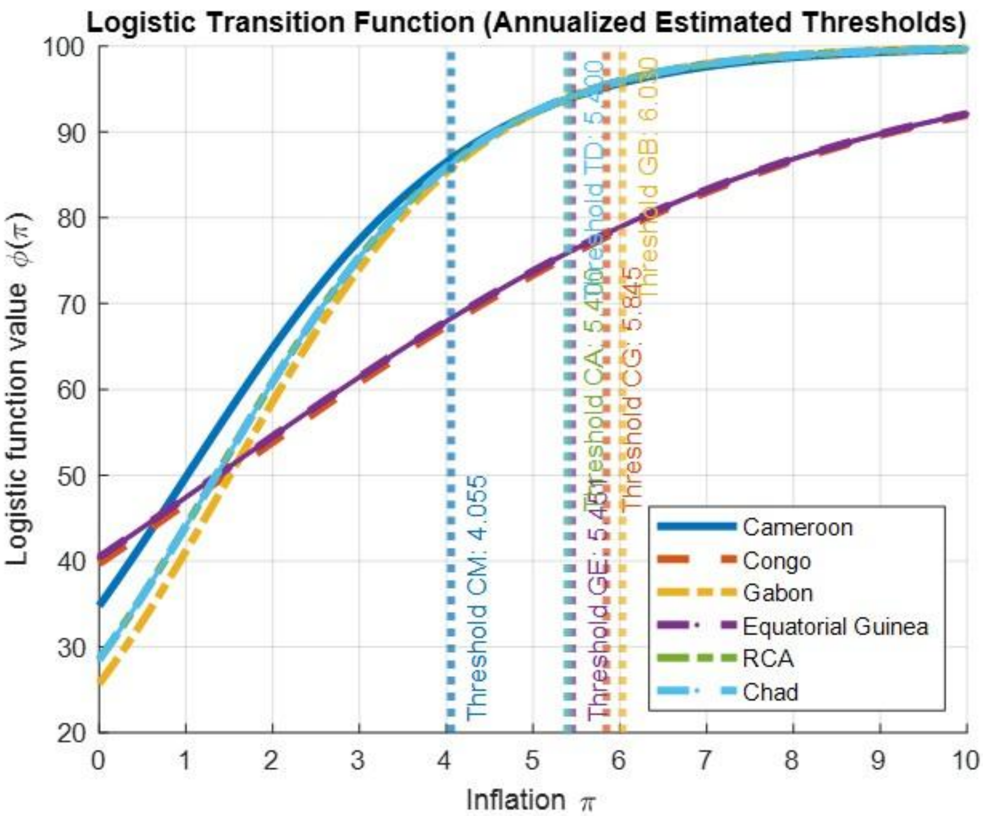
5. Results

5.1 Superiority of the nonlinear model

The marginal Likelihood for the nonlinear model (1284.614617) seems greater than the one of the linear models (1196.517093). therefore, the Log Bayes Factor (88.097524) calculated as the difference between the two confirm the superiority of the nonlinear model for CEMAC economies.

5.2 Logistic functions Slope and Thresholds Estimations by Country

The graph below summarizes the country-specific estimates of the logistic function parameters within our DSGE model: these are the inflation thresholds per country and the slope of the logistic transition curve.



The vertical dashed lines indicate the specific estimated inflation thresholds for each country, at which point the effect of monetary policy likely changes. The representation of the logistic function primarily informs us about the slope of the underlying curve. It thus appears that the state of the economy -that is, the point at which monetary policy becomes more aggressive- differs among these countries: Cameroon (4.06%), Congo (5.85%), Gabon (6.03%), Equatorial Guinea (5.45%), Central African Republic (5.40%), and Chad (5.40%). Moreover, these thresholds are relatively higher than the sub-regional convergence criterion, suggesting the possibility of raisin this criterion to allow more room for monetary policy.

Furthermore, the slope of the logistic function differs by country: Cameroon (15.53), Congo (7.15), Gabon (17.61), Equatorial Guinea (7.12), Central African Republic (12.28), and Chad (21.63). Although generally flat, it suggests varying degrees of monetary policy aggressiveness when inflation approaches thresholds, and a smooth transition that could reflect considerable inertia, particularly in Equatorial Guinea and Congo.

Finally, the country-specific intersection between the logistic function and the dashed vertical lines representing the thresholds offer five key insights for CEMAC region in terms of price stability:

- Cost-Push Inflation Trigger: When the slope of marginal cost becomes so steep that it matches or exceeds the economy's "inflation threshold," firms are forced to pass these rapidly rising costs to consumers. This marks the transition from stable production to cost-push inflation.
- Profit vs. Growth Trade-off: At the firm level, producing beyond the point where marginal costs spike may be profit-maximizing in the short term. However, if this

spike is systemic across an industry, it pushes the overall inflation rate past the threshold where it begins to erode aggregate economic growth.

- Capacity Constraints: The intersection often signals that the economy or firm has reached its maximum efficient capacity. Pushing production further causes costs (and thus prices) to rise at a rate the broader economy cannot absorb without detrimental inflationary effects.
- For Central Banks: This intersection suggests that the "real" side of the economy (production costs) is now driving the "nominal" side (inflation) past safe limits, often triggering a need for tighter monetary policy to cool demand.
- For Firms: It represents a "red zone" where further expansion may contribute to an unstable macroeconomic environment, even if individual unit sales are still marginally profitable.

5.3 Impulse Responses of Common Policy Rate, Productivity and External Shocks

The comparative analysis of Impulse Response Functions (IRFs) reveals marked heterogeneity among CEMAC countries in the face of various shocks, although general trends (return to equilibrium) remain similar.

Monetary Policy Shock Responses

Following a monetary policy shock (e.g., a rate hike), the response is consistent: Inflation and Output Gap both decrease in all countries, returning toward zero over approximately 10-20 quarters. The graphs show subtle differences in the speed of convergence and the magnitude of the initial fluctuation between countries. The "Baseline" and "Threshold" lines diverge slightly, indicating that non-linearity affects monetary policy effectiveness differently depending on the country and its specific inflation threshold.

Oil Price Shock Responses

An oil shock has varying effects: Inflation initially rises in all countries, likely due to the impact on production and transport costs, before stabilizing. Output Gap (for net oil-exporting countries - Congo, Gabon, Equatorial Guinea, Cameroon, Chad) likely see an initial improvement in economic activity, while the impact on others (CAR) may differ. The graphs show initial positive fluctuations before returning to normal.

Foreign Inflation Shock Responses

An inflation shock from trading partners is transmitted to the CEMAC economy, causing an initial rise in inflation across all countries. The impact on economic activity (output gap) is generally moderate, with minor fluctuations around zero, suggesting that imported inflation has a more direct impact on prices than on production.

Foreign Demand Shock Responses

A demand shock from trading partners stimulates both inflation and economic activity (output gap) in CEMAC countries, with initial positive fluctuations before converging toward equilibrium. Comparisons reveal that while all countries in the union react to the same shocks, the amplitude and dynamics of these responses are influenced by national specificities (such as oil dependency or inflation thresholds).

The interaction between inflation and the output gap is particularly clear here: A positive monetary policy shock (tightening) aims to reduce inflation. The graphs show that inflation (top-left lines for each country) initially decreases. Simultaneously, the output gap (top-right lines) becomes negative, indicating a slowdown or contraction in economic activity ("slack"). The decline in economic activity (negative output gap) puts downward pressure on prices, helping stabilize inflation toward zero. This is the classic Phillips Curve relationship: lower activity reduces inflationary pressures. Non-linear models ("Threshold"/"State-dependent") show that the magnitude of the inflation drop can differ from the linear model ("Baseline"), suggesting that monetary policy is more or less effective depending on whether the economy is in a high or low inflation regime at the time of the shock.

Oil Price Shocks

In this case, inflation and the output gap react differently: An oil price shock (a supply shock) increases inflation (all inflation graphs). The impact on the output gap is more varied and nuanced. For oil exporters, economic activity may initially hold steady or even increase (positive or near-zero output gap), benefiting from foreign currency inflows. For net importers, rising costs may weigh on activity (negative output gap). The effect on the output gap depends on each country's economic structure.

Foreign Demand and Foreign Inflation Shocks

Inflation and the output gap generally move in the same direction (positive correlation) following a foreign demand shock: higher demand leads to increased activity and upward pressure on prices. After a foreign Inflation Shock, Inflation is directly impacted positively, while the effect on the output gap is often weaker or transient. The graphs thus illustrate how the output gap acts as a key indicator of inflationary pressures in the economy, but the relationship between the two is complex and depends on the type of shock and the structural characteristics of each country.

The non-linear results influence the CEMAC central bank's monetary policy rule by suggesting that a "one-size-fits-all" (linear) approach is sub-optimal.

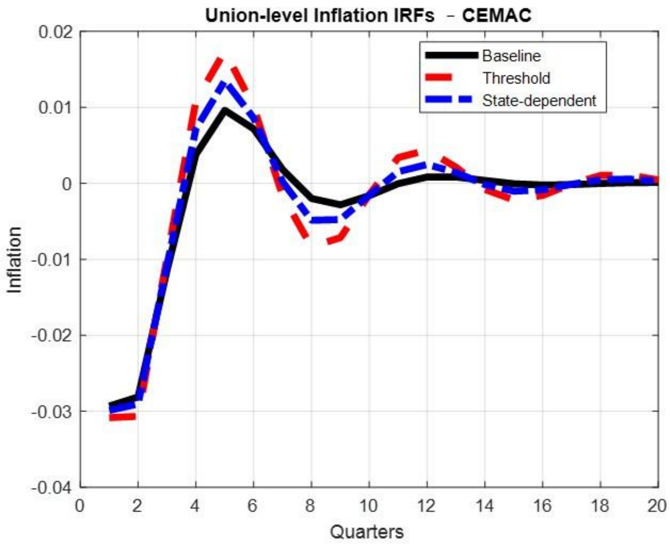
Non-Linear Monetary Policy Rule

The model suggests the adoption of a modified or "state-dependent" Taylor rule (Differentiated Response). The central bank should not react the same way to a 1% inflation deviation if the economy is in a "low inflation" regime (well below the threshold) versus a "high inflation" regime (above the threshold). The national thresholds identified in the graphs (4.5% to 7%) act as critical inflection points. If inflation exceeds these thresholds, the central bank may need to implement a more aggressive monetary policy response (a larger interest rate hike) to control inflation, as economic dynamics change beyond this point. Conversely, a more moderate response might suffice below the threshold. The "Threshold" and "State-dependent" lines in the impulse response graphs diverge from the "Baseline" (linear), visually demonstrating that the effect of monetary policy changes. The non-linear model better captures the reality of inflation dynamics in the region.

In short, the optimal monetary policy rule for CEMAC should be conditional on the prevailing inflation regime, moving away from a simple linear rule that ignores these critical tipping points.

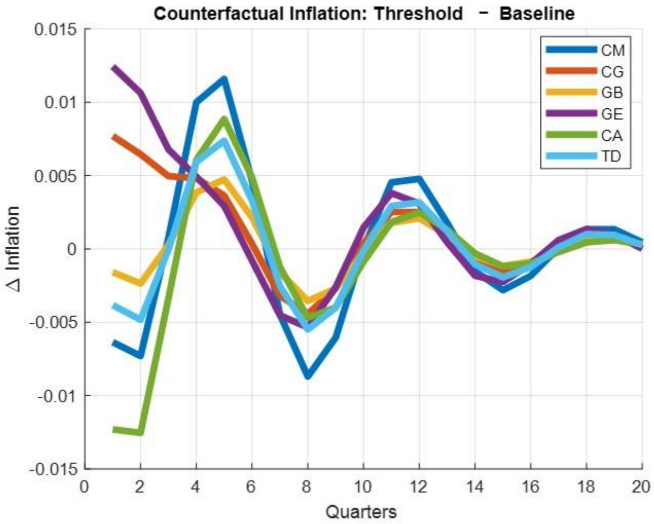
5.4 Union Level Implications of Monetary Policy Shock

The plot below shows the union-level inflation IRFs for the entire CEMAC region. It compares three different scenarios: (i) Baseline- The standard response without considering the specific thresholds; (ii) Threshold- The response when the threshold is accounted for and; (iii) State-dependent- The response based on the economy's specific state (e.g., above or below the inflation threshold).



The plot reveals the nonlinear behaviour of inflation at the regional level, highlighting the importance of the inflation threshold for accurately modelling the effects of monetary policy within the union.

5.5 Counterfactual Analysis of Inflation Response by Country (Monetary Policy Shock)



6. Policy implications

From a BEAC monetary policy perspective, the results suggest that effectiveness depends not only on the reaction to inflation gaps but also on the ability to prevent the inflationary threshold from being crossed. A monetary reaction that is too aggressive once the threshold is exceeded can generate disproportionate real costs for certain member economies. These findings argue for reinforced coordination between monetary policy and national fiscal policies, as well as the use of complementary instruments aimed at mitigating structural rigidities and stabilizing inflation expectations.

Specifically, the following key implications are underlined:

Realistic Inflation Target: the estimated real inflation thresholds for most countries (between 4.5% and 7%) are higher than the 3% target adopted by the central bank. This implies that the current target might be too low or that monetary authorities must be aware that inflation dynamics change significantly at these higher levels.

Monetary Policy Effectiveness: The effectiveness of monetary policy (its ability to control inflation and economic activity) is state-dependent, meaning it depends on whether inflation is above or below the threshold. The central bank must adjust its communication and actions based on the prevailing inflation regime.

Heterogeneity and Coordination: The existence of different inflation thresholds for each country (ranging from 4.5% in Cameroon to 7% in Chad) poses a major challenge for a single monetary policy at the union level. A policy that is optimal for one country might not be for another.

Need for Complementary Tools: The model highlights the need for complementary policies or increased coordination within the union to manage this structural heterogeneity. Specific national fiscal or structural policies might be necessary to maintain inflation around the union's target.

These results argue for a more nuanced and potentially more flexible approach to inflation management within CEMAC.

Recommendations for achieving the 3% inflation target within CEMAC, based on related research (Mvondo, 2019) include:

Reassessing the Target's Relevance: The current 3% objective is deemed significantly lower than the empirically assessed inflation threshold for the zone (estimated at approximately 4.53% on average). Monetary authorities may therefore need to re-evaluate whether this target is realistic given the region's current economic structure.

Improving the Transmission Mechanism: Research suggests that monetary policy effectiveness is compromised by the banking system's high immunity to monetary shocks. For central bank decisions (such as interest rate changes) to translate effectively into the real economy and inflation, financial reforms and financial system development are necessary to revitalize credit channels.

Policy Coordination: Close and effective coordination between monetary and fiscal policies is crucial for managing structural challenges and ensuring price stability.

Transparency and Communication: Clear communication regarding inflation targets helps anchor the inflation expectations of economic agents, contributing to the long-term credibility and effectiveness of monetary policy.

7. Conclusion

This article analyses the impact of hybrid productivity and inflationary non-linearities on monetary policy transmission within the CEMAC zone, using a multi-country DSGE model that integrates Ricardian and non-Ricardian households, physical and human capital, a non-linear Galí-Gertler Phillips (1992) curve, and a Taylor rule dependent on the inflation regime. The main innovation of the proposed framework lies in the introduction and estimation of a structural inflation threshold directly affecting the transmission of marginal costs to prices.

Bayesian estimation results indicate the existence of an inflation threshold around 5%, beyond which the pass-through from costs to inflation intensifies significantly. Below this threshold, nominal rigidities remain high and inflationary dynamics stay moderate. Above it, the Phillips curve becomes steeper, reflecting a partial de-anchoring of inflation expectations and a loss of effectiveness in traditional stabilization mechanisms. Generalized impulse response functions conditional on the inflation regime show that the transmission of a common BEAC monetary policy shock is highly dependent on the initial state of inflation. When the union's average inflation is below the estimated threshold, output and inflation responses are moderate and relatively homogeneous across countries. In contrast, when inflation exceeds the threshold, adjustments become more persistent and more costly in terms of real activity, with significantly heterogeneous responses among member countries.

This heterogeneity reflects the structural asymmetries inherent to CEMAC economies, particularly regarding production rigidities, import dependence, and initial fiscal positions. Despite a single monetary policy, some countries cross the inflationary threshold more quickly and suffer greater output losses, thus highlighting the limits of a uniform monetary rule in a monetary union characterized by differentiated economic structures. From a BEAC monetary policy perspective, the results suggest that effectiveness depends not only on the reaction to inflation gaps but also on the ability to prevent the inflationary threshold from being crossed. A monetary reaction that is too aggressive once the threshold is exceeded can generate disproportionate real costs for certain member economies. These findings argue for reinforced coordination between monetary policy and national fiscal policies, as well as the use of complementary instruments aimed at mitigating structural rigidities and stabilizing inflation expectations.

Finally, this work opens several research perspectives. A natural extension would consist of further endogenizing the inflation threshold by linking it to institutional credibility or financial conditions, or integrating climate and financial shocks that could temporarily shift the threshold. These avenues would allow for a deeper understanding of non-linear inflation mechanisms in emerging economy monetary unions and strengthen the operational relevance of DSGE models for macroeconomic policy conduct.

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Appendix A. Model details

Households

The model features two types of households: Ricardian (R) and Non-Ricardian (NR).

Ricardian Household

A fraction λ of households is Ricardian and maximizes its expected utility such that: $\max_{C_t^R, C_t^{NR}, I_t, K_{t+1}, B_{t+1}, B_t^*, H_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t U(C_t^R, C_t^N)$

Instantaneous utility is given by:

$$U(C_t^R, C_t^N) = \frac{1}{1-\sigma_C}\left(\frac{C_t^R}{C_{t-1}^R \gamma}\right)^{1-\sigma_C} - \frac{\chi}{1+\sigma_l}(H_t N_t^R)^{1+\sigma_c}$$

Where $(H_t N_t^R)$ represents effective labour.

The budget constraint is given by:

$$P_t C_t^R + P_t I_t + P_t I_t^H + B_{t+1} + \epsilon_t S_t B_{t+1}^* \leq W_t H_t N_t^R + R_t^K K_t + R_t^B B_t + \epsilon_t S_t R_t^* B_t^* + \Pi_t^R + P_t Tr_t^R$$

- ϵ_t is the nominal exchange rate.
- S_t is the price of the foreign asset in foreign currency.
- $P_t I_t$ is the household's investment expenditure.
- K_t is the capital stock, whose law of motion is given by:

$$K_{t+1} = (1-\delta)K_t + \left[1 - \frac{\phi_I}{2}\left(\frac{I_t}{I_{t-1}}\right)^2\right] I_t - \frac{\phi_\pi}{2} \max\left[\left(\frac{P_t}{P_{t-1}} - \bar{\pi}\right), 0\right]^2 I_t$$

The last term of this expression represents the dependence of inflation on the capital adjustment cost.

The dynamics of human capital are provided by an expression allowing for endogenous growth, namely:

$$H_{t+1} = (1-\delta_H)H_t + \psi_H \left(\frac{I_t^H}{H_t}\right)^{\zeta_H} H_t + \psi_{HE}(G_t^E)^{\eta_H} H_t$$

The first-order conditions for this program (with Λ_t^R as the marginal utility of income) are as follows:

Consumption

$$\Lambda_t^R = U_{C,t}^R - \beta\gamma E_t\left[U_{C,t+1}^R \frac{C_{t+1}^R}{C_t^R}\left(\frac{C_t^R}{C_{t-1}^R \gamma}\right)^{-\sigma_C}\right]$$

With $U_{C,t}^R = \left(\frac{C_t^R}{C_{t-1}^R \gamma}\right)^{-\sigma_C} \frac{1}{C_{t-1}^R \gamma}$ as the marginal utility of consumption.

Labor Supply

$$\frac{W_t}{P_t}\Lambda_t^R = U_{C,t}^R \Longrightarrow \frac{W_t}{P_t}\Lambda_t^R = \chi(N_t^R)^{\sigma_l}$$

Domestic Bonds

$$\Lambda_t^R = \beta R_t^B E_t\left[\frac{\Lambda_{t+1}^R}{\pi_{t+1}}\right]$$

Foreign Bonds

$$\epsilon_t S_t \Lambda_t^R = \beta R_t^* E_t[\epsilon_{t+1} S_{t+1} \Lambda_{t+1}^R]$$

Capital Investment

$$1 = q_t\left[1-\phi_I\left(\frac{I_t}{I_{t-1}}-1\right)\frac{I_t}{I_{t-1}}-\frac{\phi_I}{2}\left(\frac{I_t}{I_{t-1}}\right)^2-\frac{\phi_\pi}{2}\max\left[\left(\frac{P_t}{P_{t-1}}-\bar{\pi}\right),0\right]^2\right]-\beta\left[q_{t+1}\phi_I\left(\frac{I_{t+1}}{I_t}-1\right)\frac{I_{t+1}}{I_t}\right]$$

Where q_t, which stems from the work of Tobin (1969), is the marginal value of installed capital.

Capital

$$q_t = \beta E_t\left[\frac{\Lambda_{t+1}^R}{\Lambda_t^R}(R_t^K + q_{t+1}(1-\delta))\right]$$

Non-Ricardian Household

A fraction $1-\lambda$ of households is non-Ricardian and does not optimize. Its budget constraint is:

$$P_t C_t^{NR} = W_t N_t^{NR} + P_t Tr_t^{NR}$$

Firms

The Final Good Producing Firm

It operates under perfect competition and aggregates a continuum of differentiated intermediate goods ($Y_t(j)$) to produce the final good (Y_t).

Production Function

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} Y_t$$

ϵ is the elasticity of substitution between intermediate goods.

Profit Maximization

The firm maximizes its profit by choosing the demand for each intermediate good: $\max[P_t Y_t - \int_0^1 P_t(j) Y_t(j) dj]$

The first-order condition for the demand for intermediate good (j) yields the following demand function:

$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} Y_t$

The aggregate price index resulting from the zero-profit condition is:

$P_t = \left(\int_0^1 P_t(j)^{1-\epsilon} dj\right)^{\frac{1}{1-\epsilon}}$

The Intermediate Good Producing Firm

It operates under monopolistic competition and produces a differentiated good by combining capital (K), human capital (H), and labour (N), and faces Calvo-type (1983) rigidities.

Production Function

$$Y_t(j) = A_t K_t(j)^{\alpha} (H_t N_t(j))^{1-\alpha}$$

Where $(H_t N_t(j))$ represents the effective labor factor and A_t represents the hybrid technology.

Cost Minimization

Subject to the production function constraint, the firm chooses $K_t(j)$ and $N_t(j)$, given factor costs:

$$\min_{K_t(j), N_t(j)} [R_t^K K_t(j) + W_t N_t(j)]$$

Hence the marginal cost:

$$mc_t = \frac{1}{A_t}\left(\frac{R_t^K}{\alpha}\right)^{\alpha}\left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}$$

Profit Maximization and Calvo Principle

Following Calvo (1983), firms optimize their prices with probability $1-\xi$ and index them to the previous period with probability ξ. The first-order condition of the optimization program is:

$$E_t \sum_{k=0}^{\infty} (\beta\xi)^k \Lambda_{t,t+k} [(P_t^* - \epsilon \frac{P_{t,t+k}}{P_t^*} mc_{t+k}) Y_{t+k}(j)] = 0$$

Resulting in the hybrid Phillips curve:

$$\pi_{t,j} = \frac{(1-\xi)(1-\beta\xi)}{\xi} mc_t + \beta E_t \pi_{t+1} + \chi_p \pi_{t-1} + \epsilon_{t,j}^{\pi_j}$$

With:

- χ_p as the degree of Calvo (1983) indexation.
- $\kappa_j^{low} = \frac{(1-\xi)(1-\beta\xi)}{\xi}$
- $\kappa_j = \kappa_j^{low} + (\kappa_j^{high} - \kappa_j^{low}) F(A_{t,j}, \bar{A}_j)$
- $F(A_{t,j}, \bar{A}_j) = \frac{1}{1+\exp[R\&D_{t,j} - \bar{A}_j]}$

The R&D Sector Firm

The R&D sector creates new technological products, thus contributing to the dynamics of the hybrid productivity (A) given by:

$$A_{t,j} = A_{t,j}^{Foreign} A_{t,j}^{Domestic}$$

And therefore:

$$\log(A_{t,j}) = \log(A_{t,j}^{Foreign}) + \log(A_{t,j}^{Domestic})$$

Thus, $A_t^{Foreign}$ is an autoregressive process that benefits from foreign productivity dynamics and is considered the exogenous part, while $A_t^{Domestic}$ follows endogenous evolution. Their expressions are provided by:

$$A_{t,j}^{Foreign} = \epsilon_{t,j}^{Foreign} A_{t-1,j}^{\rho_{A,Foreign}}$$

$$A_{t,j}^{Domestic} = F(R\&D_{t,j}, A_{t-1,j})$$

The State

$$P_t G_t + B_{t+1} + P_t Tr_t = Tax_t + R_t^B B_t + Tr_{t+1}$$

The Central Bank

The standard Taylor rule (1993) is given by:

$$R_t^B = \bar{R} \left(\frac{P_t/P_{t-1}}{\bar{\pi}}\right)^{\phi_\pi} \left(\frac{Y_t}{Y_t^*}\right)^{\phi_y}$$

Open Economy

Net exports depend on the real effective exchange rate ($Q_t = \frac{\epsilon_t P_t^*}{P_t}$) and relative demand conditions:

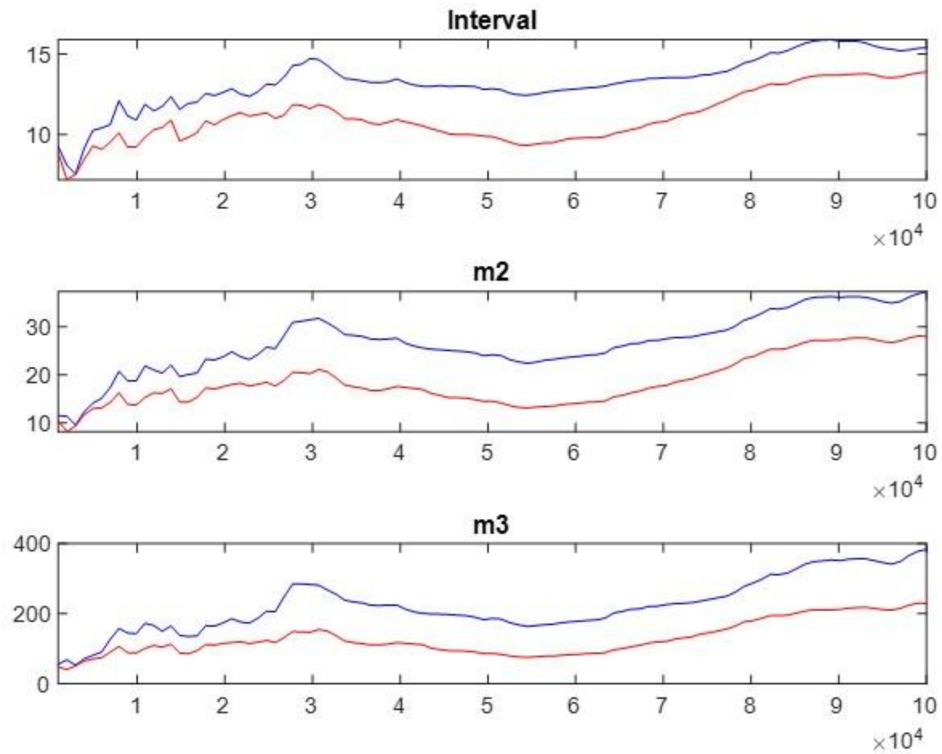
$$NX_t = f(Q_t, Y_t, Y_t^*) = \psi_{NX} Y_t^* - \psi_{NX,Y} Y_t + Q, Q_t$$

The resource constraint is:

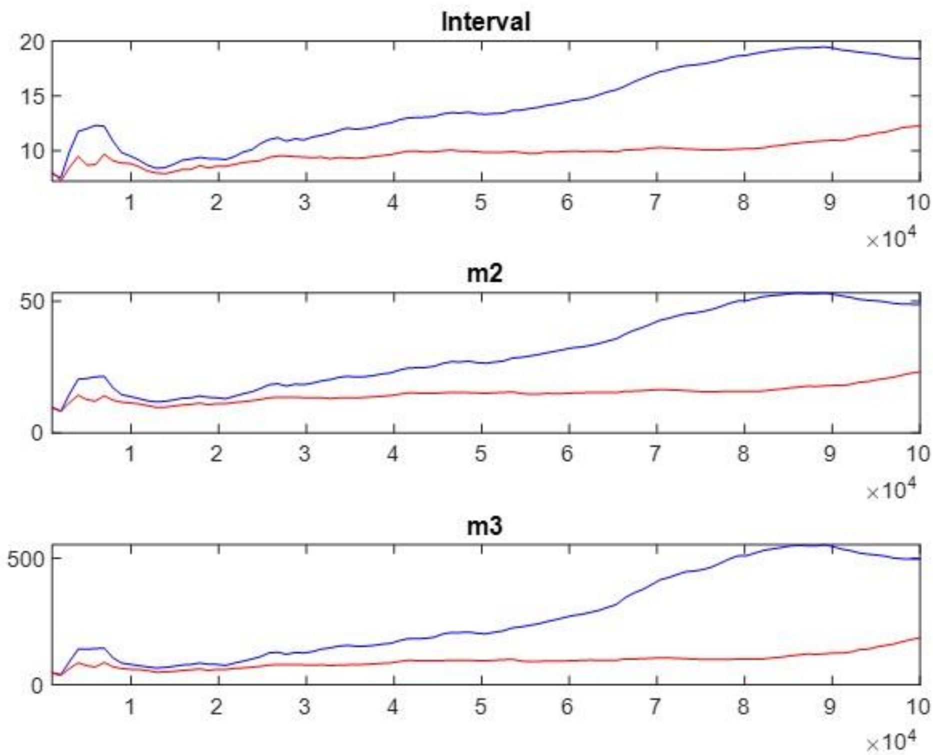
$$Y_t = C_t^R + C_t^{NR} + I_t + I_t^H + G_t + NX_t$$

Appendix B. Multivariate convergence diagnostic

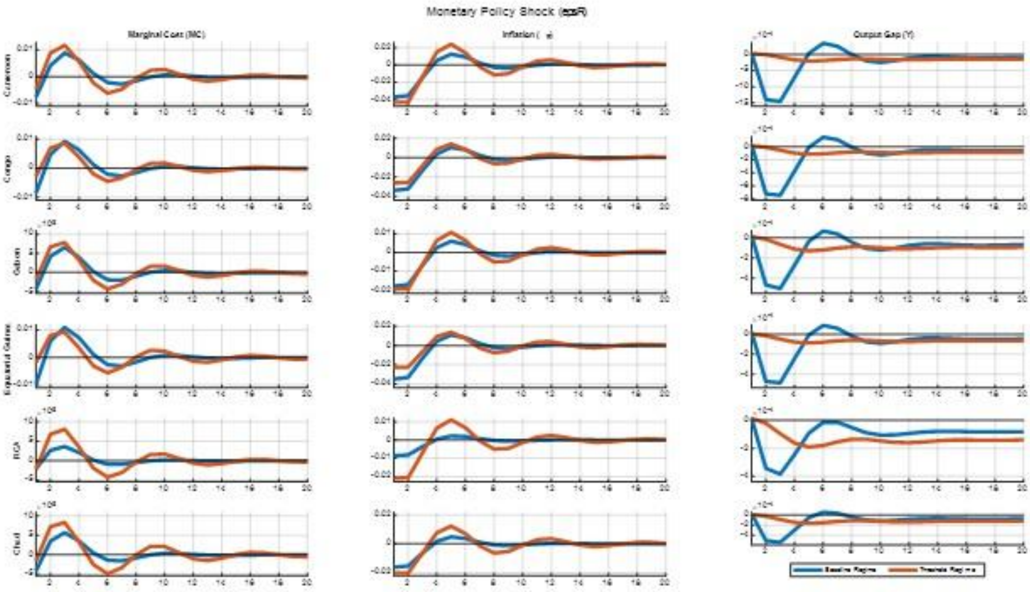
LINEAR MODEL

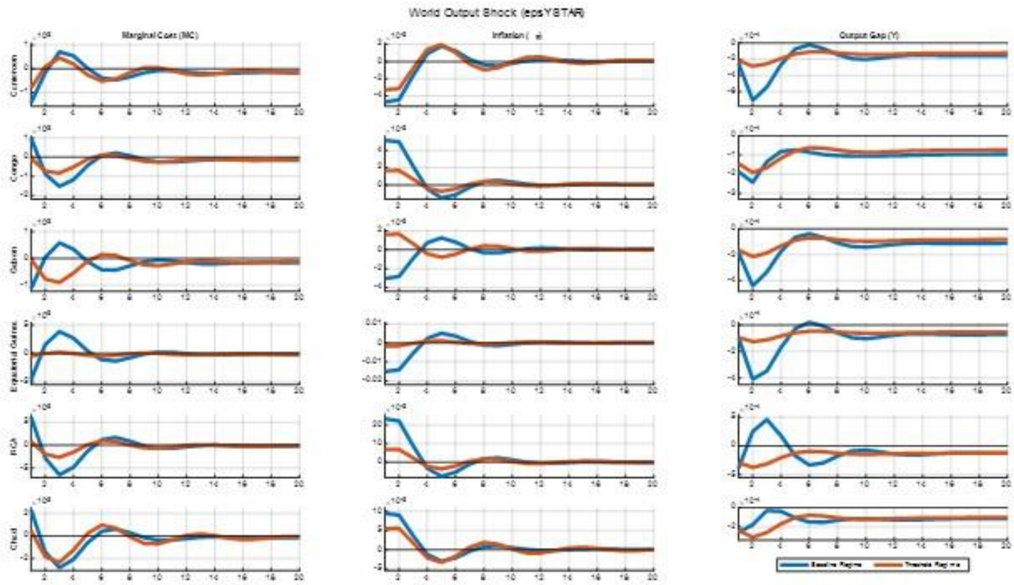


NONLINEAR MODE

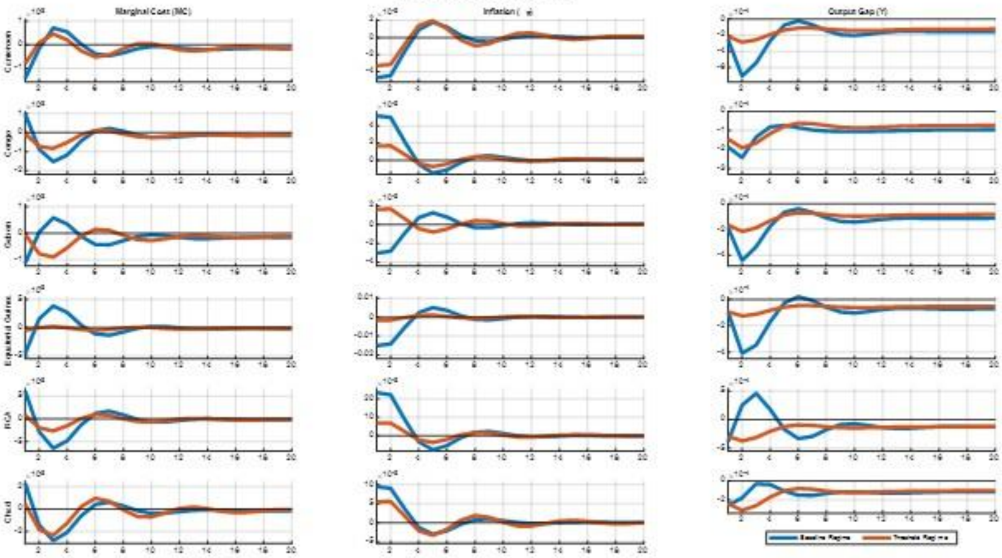


Appendix C. Impulse Response Functions for specific shocks

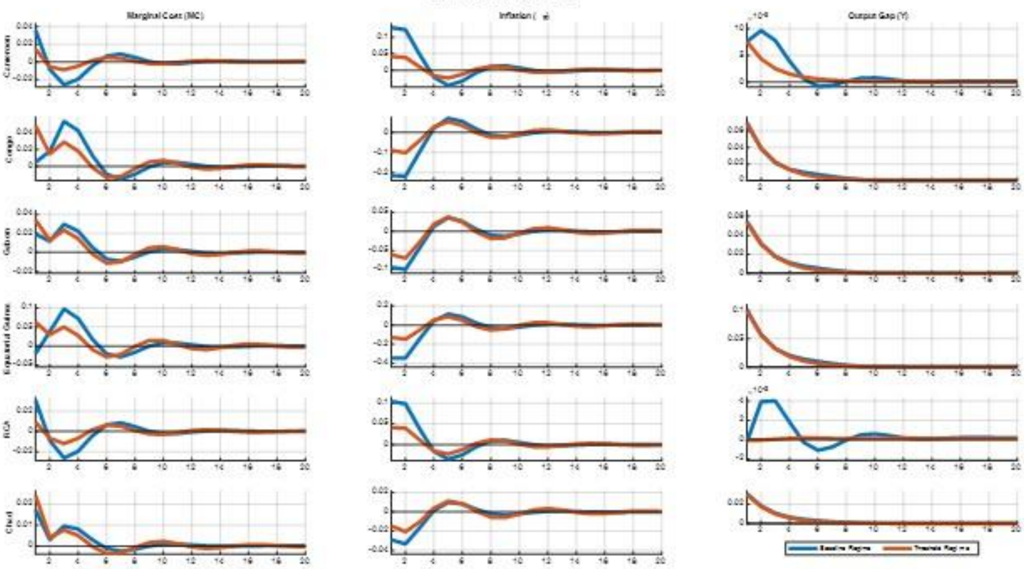




World Output Shock (epsYSTAR)



Oil Price Shock (epsGPOIL)



Appendix D. Impulse Response Functions of hybrid productivity shocks